1   # Algebra I

2

3   **Introduction**

4   The following Traditional and Integrated Pathways lay out two sequences of courses

5   that include the "College and Career Readiness" standards of the higher mathematics

6   CA CCSSM. These are the standards deemed appropriate for all students to learn in

7   preparation for pursuing a college education or a career in a technology-infused 21st

8   Century. Most of the additional "plus" (+) standards are treated in courses beyond

9   these pathways and are intended to be learned by those students who wish to pursue

10   higher-level mathematics courses or college majors in a STEM field.

11

12   These pathways and the two suggested "fourth courses" presented in the framework do

13   not represent the only courses available for high school students. The higher

14   mathematics CA CCSSM are designed in part as a menu of standards from which

15   educators can create customized courses. Thus, for example, a school or district may

16   opt to create a mathematics course based on certain CA CCSSM and the Career

17   Technical Education Model Curriculum Standards, or they may opt to create a

18   Mathematical Modeling course using starred (★) standards (see the Appendix,

19   "Mathematical Modeling," for more information). Overall, schools and districts have a

20   choice in their course offerings at the high school level and are welcome to take

21   advantage of the focus on real-world applications of the higher mathematics standards

22   and the overall flexibility of the CA CCSSM.

23

24   The main purpose of Algebra I is to develop students' fluency with linear, quadratic and

25   exponential functions. The critical areas of instruction involve deepening and extending

26   students' understanding of linear and exponential relationships by contrasting them with

27   each other and by applying linear models to data that exhibit a linear trend. In addition,

28   students engage in methods for analyzing, solving, and using exponential and quadratic

29   functions. Some of the overarching ideas in the Algebra I course include: the notion of

30   function, solving equations, rates of change and growth patterns, graphs as

31   representations of functions, and modeling.

32

33   The standards in the Traditional Pathway Algebra I course come from the following

34   conceptual categories: Modeling, Functions, Number and Quantity, Algebra, and

35   Statistics and Probability. The content of the course will be expounded on below

36   according to these conceptual categories, but teachers and administrators alike should

37   note that the standards are not listed here in the order in which they should be taught.

38   Moreover, the standards are not simply topics to be checked off a list during isolated

39   units of instruction, but rather content that should be present throughout the school year

40   through rich instructional experiences.

41

42   **What Students learn in Algebra I**

43   **Overview**

44   In Algebra I, students use reasoning about structure to define and make sense of

45   rational exponents and explore the algebraic structure of the rational and real number

46   systems. They understand that numbers in real world applications often have units

47   attached to them, that is, they are considered *quantities*. Students' work with numbers

48   and operations throughout elementary and middle school has led them to an

49   understanding of the structure of the number system; now, students explore the

50   structure of algebraic expressions and polynomials. They see that certain properties

51   must persist when working with expressions that are meant to represent numbers, now

52   written in an abstract form involving variables. When two expressions with overlapping

53   domains are set equal to each other, resulting in an equation, there is an implied

54   solution set (be it empty or non-empty), and students not only refine their techniques for

55   solving equations and finding the solution set, but they can clearly explain the algebraic

56   steps they used to do so.

57

58   Students began their exploration of linear equations in middle school. They first

59   connected proportional equations ($y = kx, k \neq 0$) to graphs, tables and real-world

60   contexts, and then moved towards an understanding of general linear equations

61   ($y = mx + b, m \neq 0$) and their graphs. In Algebra I, students extend this knowledge to

62   working with absolute value equations, linear inequalities, and systems of linear

63   equations. After learning a more precise definition of function in this course, students

64   examine this new idea in the familiar context of linear equations (for example, by seeing

65   the solution of a linear equation as solving $f(x) = g(x)$ for two linear functions f and g).

66

67   Students continue building their understanding of functions beyond linear ones by

68   investigating tables, graphs, and equations that build on previous understandings of

69   numbers and expressions. They make connections between different representations

70   of the same function. They learn to build functions in a modeling context, and solve

71   problems related to the resulting functions. Note that the focus in Algebra I is on linear,

72   simple exponential, and quadratic equations.

73

74   Finally, students extend their prior experiences with data, using more formal means of

75   assessing how a model fits data. Students use regression techniques to describe

76   approximately linear relationships between quantities. They use graphical

77   representations and knowledge of the context to make judgments about the

78   appropriateness of linear models. With linear models, they look at residuals to analyze

79   the goodness of fit.

80

81   **Examples of Key Advances from Grades K–8**

82   • Having already extended arithmetic from whole numbers to fractions (grades 4–

83   6) and from fractions to rational numbers (grade 7), students in grade 8

84   encountered particular irrational numbers such as $\sqrt{5}$ and π. In Algebra I,

85   students will begin to understand the real number *system*. (For more on the

86   extension of number systems, see page 58 of the CCSSI document, *Common*

87   *Core State Standards for Mathematics*).

88   • Students in middle grades worked with measurement units, including units

89   obtained by multiplying and dividing quantities. In Algebra I, students apply these

90   skills in a more sophisticated fashion to solve problems in which reasoning about

91   units adds insight (N-Q).

92   • Algebraic themes beginning in middle school continue and deepen during high

93   school. As early as grades 6 and 7, students began to use the properties of

94   operations to generate equivalent expressions (6.EE.3, 7.EE.1). By grade 7, they

95   began to recognize that rewriting expressions in different forms could be useful in

96   problem solving (7.EE.2). In Algebra I, these aspects of algebra carry forward as

97   students continue to use properties of operations to rewrite expressions, gaining

98   fluency and engaging in what has been called "mindful manipulation."

99   • Students in grade 8 extended their prior understanding of proportional

100   relationships to begin working with functions, with an emphasis on linear

101   functions. In Algebra I, students will master linear and quadratic functions.

102   Students encounter other kinds of functions to ensure that general principles are

103   perceived in generality, as well as to enrich the range of quantitative relationships

104   considered in problems.

105   • Students in grade 8 connected their knowledge about proportional relationships,

106   lines and linear equations (8.EE.5, 6). In Algebra I, students solidify their

107   understanding of the analytic geometry of lines. They understand that in the

108   Cartesian coordinate plane:

109   o The graph of any linear equation in two variables is a line.

110   o Any line is the graph of a linear equation in two variables.

111   • As students acquire mathematical tools from their study of algebra and functions,

112   they apply these tools in statistical contexts (e.g., S-ID.6). In a modeling context,

113   they might informally fit a quadratic function to a set of data, graphing the data

114   and the model function on the same coordinate axes. They also draw on skills

115   they first learned in middle school to apply basic statistics and simple probability

116   in a modeling context. For example, they might estimate a measure of center or

117   variation and use it as an input for a rough calculation.

118   • Algebra I techniques open a huge variety of word problems that can be solved

119   that were previously inaccessible or very complex in grades K–8. This expands

120   problem solving from grades K–8 dramatically.

121

122   **Connecting Standards for Mathematical Practice and Content**

123   The Standards for Mathematical Practice apply throughout each course and, together

124   with the content standards, prescribe that students experience mathematics as a

125   coherent, relevant, and meaningful subject that makes use of their ability to make sense

126   of problem situations. The Standards for Mathematical Practice (MP) represent a

127   picture of what it looks like for students to *do mathematics* and, to the extent possible,

128   content instruction should include attention to appropriate practice standards. There are

129   ample opportunities for students to engage in each mathematical practice in Algebra I;

130   the table below offers some general examples.

131

Standards for Mathematical Practice *Students…*	Examples of each practice in Algebra I
MP1. *Make sense of problems and persevere in solving them.*	Students learn that often patience is required to fully understand what a problem is asking. They discern between what information is useful, and what is not. They expand their repertoire of expressions and functions that can used to solve problems.
MP2. *Reason abstractly and quantitatively.*	Students extend their understanding of slope as the rate of change of a linear function to understanding that the average rate of change of any function can be computed over an appropriate interval.
MP3. *Construct viable arguments and critique the reasoning of others.*	Students reason through the solving of equations, recognizing that solving an equation is more than simply a matter of rote rules and steps. They use language such as "if… then..." when explaining their solution methods and

***Students build proofs by induction and proofs by contradiction. CA 3.1** (for higher mathematics only).*	provide justification.
MP4. Model with mathematics.	Students also discover mathematics through experimentation and examining patterns in data from real world contexts. Students apply their new mathematical understanding of exponential, linear and quadratic functions to real-world problems.
MP5. Use appropriate tools strategically.	Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret the result. They construct diagrams to solve problems.
MP6. Attend to precision.	Students begin to understand that a *rational number* has a specific definition, and that *irrational numbers* exist. They make use of the definition of *function* when deciding if an equation can describe a function by asking, "Does every input value have exactly one output value?"
MP7. Look for and make use of structure.	Students develop formulas such as $(a \pm b)^2 = a^2 \pm 2ab + b^2$ by applying the distributive property. Students see that the expression $5 + (x - 2)^2$ takes the form of "5 plus 'something' squared", and so that expression can be no smaller than 5.
MP8. Look for and express regularity in repeating reasoning.	Students see that the key feature of a line in the plane is an equal difference in outputs over equal intervals of inputs, and that the result of evaluating the expression $\frac{y_2 - y_1}{x_2 - x_1}$ for points on the line is always equal to a certain number m. Therefore, if (x, y) is a generic point on this line, the equation $m = \frac{y - y_1}{x - x_1}$ will give a general equation of that line.

132

133   MP 4 holds a special place throughout the higher mathematics curriculum, as Modeling

134   is considered its own conceptual category. Though the Modeling category has no

135   specific standards listed within it, the idea of using mathematics to model the world

136   pervades all higher mathematics courses and should hold a high place in instruction.

137   Readers will see some standards marked with a star symbol (★) to indicate that they are

138   *modeling standards*, that is, they present an opportunity for applications to real-world

139   modeling situations more so than other standards. In the description of the Algebra I

140   Content Standards below, Modeling comes first to emphasize its important in the higher

141   mathematics curriculum.

142 Examples of places where specific Mathematical Practice standards can be

143 implemented in the Algebra I standards will be noted in parentheses, with the

144 standard(s) indicated.

145

146 **Algebra I Content Standards by Conceptual Category**

147 The Algebra I course is organized by conceptual category, domains, clusters, and then

148 standards. Below, the overall purpose and progression of the standards included in

149 Algebra I are described according to these conceptual categories. Standards that are

150 considered to be new to secondary grades teachers will be discussed in more depth

151 than others.

152

153   **Conceptual Category: Modeling**

154 Throughout the higher mathematics CA CCSSM standards, certain standards are

155 marked with a (⋆) symbol to indicate that they are considered modeling standards.

156 Modeling at the higher mathematics level goes beyond the simple application of

157 previously constructed mathematics to real-world problems. True modeling begins with

158 students asking a question about the world around them, and mathematics is then

159 constructed in the process of attempting to answer the question. When students are

160 presented with a real world situation and challenged to ask a question, all sorts of new

161 issues arise: which of the quantities present in this situation are known and unknown?

162 Can I make a table of data? Is there a functional relationship in this situation? Students

163 need to decide on a solution path, which may need to be revised. They make use of

164 tools such as calculators, dynamic geometry software, or spreadsheets. They try to use

165   previously derived models (e.g. linear functions) but may find that a new equation or

166   function will apply. They may see that solving an equation arises as a necessity when

167   trying to answer their question, and that oftentimes the equation arises as the specific

168   instance of knowing the output value of a function at an unknown input value.

169

170   Modeling problems have an element of being genuine problems, in the sense that

171   students care about answering the question under consideration. In modeling,

172   mathematics is used as a tool to answer questions that students really want answered.

173   This will be a new approach for many teachers and will be challenging to implement, but

174   the effort will produce students who can appreciate that mathematics is relevant to their

175   lives. From a pedagogical perspective, modeling gives a concrete basis from which to

176   abstract the mathematics and often serves to motivate students to become independent
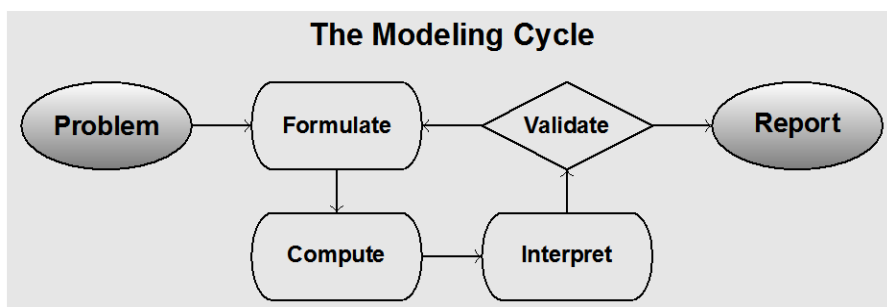
177   learners.

178
179   Figure 1: The modeling cycle. Students examine a problem and formulate a *mathematical model* (an
180   equation, table, graph, etc.), compute an answer or rewrite their expression to reveal new information,
181   interpret their results, validate them, and report out.

182

183   Throughout the Algebra I chapter, the included examples will be framed as much as

184   possible as modeling situations, to serve as illustrations of the concept of mathematical

185   modeling. The big ideas of linear and exponential functions, graphing, solving

186   equations, and rates of change will be explored through this lens. The reader is

187   encouraged to consult the Appendix on Mathematical Modeling for a further discussion

188   of the modeling cycle and how it is integrated into the higher mathematics curriculum.

189

190   **Conceptual Category: Functions**

191   Functions describe situations where one quantity determines another. For example, the

192   return on $10,000 invested at an annualized percentage rate of 4.25% is a function of

193   the length of time the money is invested. Because we continually make theories about

194   dependencies between quantities in nature and society, functions are important tools in

195   the construction of mathematical models. In school mathematics, functions usually

196   have numerical inputs and outputs and are often defined by an algebraic expression.

197   For example, the time in hours it takes for a car to drive 100 miles is a function of the

198   car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship

199   algebraically and defines a function whose name is T.

200

201   The set of inputs to a function is called its domain. We often assume the domain to be

202   all inputs for which the expression defining a function has a value, or for which the

203   function makes sense in a given context. When describing relationships between

204   quantities, the defining characteristic of a *function* is that the input value determines the

205   output value, or equivalently, that the output value depends upon the input value.

206

207   A function can be described in various ways, such as by a graph (e.g., the trace of a

208   seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;"

209   by an assignment, such as the fact that each individual is given a unique Social Security

210   Number; by an algebraic expression like $f(x) = a + bx$; or by a recursive rule, such as

211    $f(n + 1) = f(n) + b, f(0) = a$. The graph of a function is often a useful way of

212   visualizing the relationship that the function models, and manipulating a mathematical

213   expression for a function can shed light on the function's properties.

214
215   **Interpreting Functions**   **F-IF**

216   **Understand the concept of a function and use function notation.** [Learn as general principle; focus on linear
217   and exponential and on arithmetic and geometric sequences]

218   1.   Understand that a function from one set (called the domain) to another set (called the range)
219   assigns to each element of the domain exactly one element of the range. If f is a function and x is
220   an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The
221   graph of f is the graph of the equation $y = f(x)$.
222   2.   Use function notation, evaluate functions for inputs in their domains, and interpret statements that
223   use function notation in terms of a context.
224   3.   Recognize that sequences are functions, sometimes defined recursively, whose domain is a
225   subset of the integers. *For example, the Fibonacci sequence is defined recursively by*
226   *f(0) = f(1) = 1, f(n + 1) = f(n) + f(n − 1) for n ≥ 1.*
227

228   **Interpret functions that arise in applications in terms of the context.** [Linear, exponential, and quadratic.]

229   4.   For a function that models a relationship between two quantities, interpret key features of graphs
230   and tables in terms of the quantities, and sketch graphs showing key features given a verbal
231   description of the relationship. *Key features include: intercepts; intervals where the function is*
232   *increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end*
233   *behavior; and periodicity.* ★
234   5.   Relate the domain of a function to its graph and, where applicable, to the quantitative relationship
235   it describes. *For example, if the function h gives the number of person-hours it takes to assemble*
236   *n engines in a factory, then the positive integers would be an appropriate domain for the function.*
237   ★
238   6.   Calculate and interpret the average rate of change of a function (presented symbolically or as a
239   table) over a specified interval. Estimate the rate of change from a graph. ★
240
241

242   While the grade eight standards call for students to work informally with functions, in

243   Algebra I they begin to refine their understanding and use the formal mathematical

244   language of functions. Standards F-IF.1-9 deal with understanding the concept of a

245   function, interpreting characteristics of functions in context, and representing functions

246   in different ways (MP.6). In F.IF.1-3, students learn the language of functions and that a

247   function has a domain that must be specified and a corresponding range. For instance,

248   the function f where $f(n) = 4(n − 2)^2$, defined for n an integer, is a different function

249   than the function g where $g(x) = 4(x - 2)^2$ and g is defined for all real numbers x.

250   Students make the connection between the graph of the equation $y = f(x)$ and the

251   function itself, namely, that the coordinates of any point on the graph represent an input

252   and output, expressed as $(x, f(x))$, and understand that the graph is a *representation* of

253   a function. They connect the domain and range of a function to its graph (F.IF.5). Note

254   that there is neither an exploration of the notion of *relation vs. function* nor the *vertical*

255   *line test* in the CA CCSSM. This is by design. The core question when investigating

256   functions is: "Does each element of the domain correspond to exactly one element of

257   the range?"

258

259   Standard F.IF.3 represents a topic new to the traditional Algebra I course, that of

260   *sequences*. Sequences are functions with a domain consisting of a subset of the

261   integers. Back in grades four and five, students began to explore number patterns, and

262   this work led into a full progression of ratios and proportional relationships in grades six

263   and seven. Patterns are examples of sequences, and the work here is intended to

264   formalize and extend students' earlier understandings. For a simple example consider

265   the sequence $4, 7, 10, 13, 16, \dots$ which might be described as a "plus 3 pattern" because

266   terms are computed by adding 3 to the previous term. If we decided that 4 is the first

267   term of the sequence then we can make a table, a graph, and eventually a recursive

268   rule for this sequence: $f(1) = 4, f(n + 1) = f(n) + 3$ for $n \geq 2$. Of course, this

269   sequence can also be described with explicit formula $f(n) = 3n + 1$ for $n \geq 1$. Notice

270   that the domain is included in the description of the rule. In Algebra I, students should

271   have opportunities to work with linear, quadratic and exponential sequences, and to

272   interpret the parameters of the expressions defining the terms of the sequence when

273   they arise in context.

274
275   **Interpreting Functions**   **F-IF**

276   **Analyze functions using different representations.** [Linear, exponential, quadratic, absolute value, step, piecewise-
277   defined.]

278   7.   Graph functions expressed symbolically and show key features of the graph, by hand in simple
279   cases and using technology for more complicated cases.   ★
280   a.   Graph linear and quadratic functions and show intercepts, maxima, and minima. ★
281   b.   Graph square root, cube root, and piecewise-defined functions, including step functions and
282   absolute value functions. ★
283   e.   Graph exponential and logarithmic functions, showing intercepts and end behavior, and
284   trigonometric functions, showing period, midline, and amplitude. ★
285   8.   Write a function defined by an expression in different but equivalent forms to reveal and explain
286   different properties of the function.
287   a.   Use the process of factoring and completing the square in a quadratic function to show zeros,
288   extreme values, and symmetry of the graph, and interpret these in terms of a context.
289   b.   Use the properties of exponents to interpret expressions for exponential functions. *For*
290   *example, identify percent rate of change in functions such as* $y = (1.02)^t$*,* $y = (0.97)^t$*,*
291                 $y = (1.01)^{12t}$*, and* $y = (1.2)^{t/10}$*, and classify them as representing exponential growth or decay.*
292   9.   Compare properties of two functions each represented in a different way (algebraically,
293   graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one*
294   *quadratic function and an algebraic expression for another, say which has the larger maximum.*
295

296   In standards F-IF.7-9, students represent functions with graphs and identify key features

297   in the graph. In Algebra I, linear, exponential and quadratic functions are given

298   extensive treatment since they themselves have their own group of standards (the F-LE

299   standards) dedicated to them. Students are expected to develop fluency with only

300   linear, exponential and quadratic functions in Algebra I, which includes the ability to

301   graph them by hand.

302

303   In this set of three standards, students represent the same function algebraically in

304   different forms and interpret these differences in terms of the graph or context. For

305   instance, students may easily see that the graph of the equation $f(x) = 3x^2 + 9x + 6$

306   crosses the y-axis at $(0,6)$ since the terms with x in them are simply 0 when $x = 0$, but

307   they then factor the expression defining f to obtain $f(x) = 3(x + 2)(x + 1)$, easily

308   revealing that the function crosses the x-axis at $(-2, 0)$ and $(-1, 0)$, since this is where

309    $f(x) = 0$ (MP.7).

310

311   **Building Functions**   **F-BF**

312   **Build a function that models a relationship between two quantities.** [For F.BF.1, 2, linear, exponential, and
313   quadratic.]

314   1.   Write a function that describes a relationship between two quantities.   ★
315   a.   Determine an explicit expression, a recursive process, or steps for calculation from a
316   context.   ★
317   b.   Combine standard function types using arithmetic operations. *For example, build a function*
318   *that models the temperature of a cooling body by adding a constant function to a decaying*
319   *exponential, and relate these functions to the model.* ★
320   2.   Write arithmetic and geometric sequences both recursively and with an explicit formula, use them
321   to model situations, and translate between the two forms. ★

322

323   **Build new functions from existing functions.** [Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear
324   only.]

325   3.   Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific
326   values of k (both positive and negative); find the value of k given the graphs. Experiment with
327   cases and illustrate an explanation of the effects on the graph using technology. *Include*
328   *recognizing even and odd functions from their graphs and algebraic expressions for them.*
329   4.   Find inverse functions.
330   a.   Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an
331   expression for the inverse.

332

333   Knowledge of functions and expressions is only part of the complete picture. One must

334   be able to understand a given situation and apply function reasoning to model how

335   quantities change together. Often, the function created sheds light on the situation at

336   hand; one can make predictions of future changes, for example. This is the content of

337   standards F-BF.1 and F-BF.2 (starred to indicate they are Modeling standards). A

338   strong connection exists between standard F.BF.1 and standard A.CDE.2, which

339   discusses creating equations. The following example shows that students can create

340   functions based on prototypical ones.

341

Exponential Growth. When a quantity grows with time by a multiplicative factor greater than 1, it is said the quantity grows exponentially. Hence, if an initial population of bacteria, P_0, doubles each day, then after t days, the new population is given by $$P(t) = P_0 2^t.$$ This expression can be generalized to include different growth rates, r, as in $P(t) = P_0 r^t$. The following example illustrates the type of problem that students can face after they have worked with basic exponential functions like these. **Example.** On June 1, a fast growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area covered by the algae doubles every day. If it continues to grow unabated, the lake will be totally covered and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30.	**Possible Questions to Ask:** a. When will the lake be covered halfway? b. Write an equation that represents the percentage of the surface area of the lake that is covered in algae as a function of time (in days) that passes since the algae was introduced into the lake. **Solution and Comment.** a. Since the population doubles each day, and since the entire lake is covered by June 30, this implies that half the lake was covered on June 29. b. If $P(t)$ represents the *percentage* of the lake covered by the algae, then we know that $P(29) = P_0 2^{29} = 100$ (note that June 30 corresponds to $t = 29$). Therefore, we can solve for the initial percentage of the lake covered, $P_0 = \frac{100}{2^{29}} \approx 1.86 \times 10^{-7}$. The equation for the percentage of the lake covered by algae at time t is therefore $P(t) = (1.86 \times 10^{-7})2^t$.

342

343   As mentioned earlier, new to the Algebra I course in California is the study of arithmetic

344   and geometric sequences, written both explicitly and recursively (F-BF.2). Often when

345   presented with a sequence, students can manage to find the recursive pattern of the

346   sequence, i.e. how the sequence changes from term to term. For instance, a simple

347   doubling pattern can lead to an exponential expression of the form $a2^n$, for $n \geq 0$.

348   Ample experience with linear and exponential functions—which show equal differences

349   over equal intervals and equal ratios over equal intervals, respectively—can provide

350   students with tools for finding explicit rules for sequences. Investigating the simple

351   sequence of squares, $f(n) = n^2$, $n \geq 1$ provides a prototype for other basic quadratic

352   sequences. Diagrams, tables, and graphs can help students make sense of the different

353   rates of growth all three sequences exhibit.

Example: *Cellular Growth.* Populations of cyanobacteria can double every 6 hours under ideal conditions, at least until the nutrients in its supporting culture are depleted. This means a population of 500 such bacteria would grow to 1000 in the first 6-hour period, 2000 in the second 6-hour period, 4000 in the third 6-hour period, etc. Evidently, if n represents the number of 6-hour periods from the start, the population at that time $P(n)$ satisfies $P(n) = 2 \cdot P(n-1)$. This is a *recursive* formula for the sequence $P(n)$, which	gives the population at a given time period n in terms of the population at time period $n-1$. To find a closed, *explicit*, formula for $P(n)$, students can reason that $$P(0) = 500, P(1) = 2 \cdot 500, P(2) = 2 \cdot 2 \cdot 500, P(3)$$ $$= 2 \cdot 2 \cdot 2 \cdot 500, ...$$ A pattern emerges, that $P(n) = 2^n \cdot 500$. In general, if an initial population P_0 grows by a factor $r > 1$ over a fixed time period, then the population after n time periods is given by $P(n) = P_0 r^n$.

354
355   The content of standard F-BF.3 has typically been left to later courses. In Algebra I, the

356   focus is on linear, exponential and quadratic functions. Even and odd functions are

357   addressed in later courses. In keeping with the theme of the input-output interpretation

358   of a function, students should work towards developing an understanding of the effect

359   on the output of a function under certain transformations, such as in the table below:

Expression	Interpretation
$f(a+2)$	The output when the input is 2 greater than a
$f(a)+3$	3 more than the output when the input is a
$2f(x)+5$	5 more than twice the output of f when the input is x

360
361   Such understandings can help in seeing the effect of transformations on the graph of a

362   function, and in particular, can aid in understanding why it appears that the effect on the

363   graph is the opposite to the transformation on the variable (e.g. the graph of $y = f(x +$

364   2) is the graph of f shifted 2 units to the left, not to the right).

365

366   Also new to the Algebra I course is standard F-BF.4, in which students find inverse

367   functions in simple cases. For example, an Algebra I student might solve the equation

368   $C = \frac{9}{5}F + 32$ for F. The student starts with this formula, showing how Celsius

369   temperature is a function of Fahrenheit temperature, and by solving for F finds the

370   formula for the inverse function. This is a contextually appropriate way to find the

371   expression for an inverse function, in contrast with the practice of simply swapping x

372   and y in an equation and solving for y.

373

374   **Linear, Quadratic, and Exponential Models**   **F-LE**

375   **Construct and compare linear, quadratic, and exponential models and solve problems.**

376   1.   Distinguish between situations that can be modeled with linear functions and with exponential
377   functions. ★
378   a.   Prove that linear functions grow by equal differences over equal intervals, and that
379   exponential functions grow by equal factors over equal intervals. ★
380   b.   Recognize situations in which one quantity changes at a constant rate per unit interval
381   relative to another. ★
382   c.   Recognize situations in which a quantity grows or decays by a constant percent rate per unit
383   interval relative to another. ★
384   2.   Construct linear and exponential functions, including arithmetic and geometric sequences, given
385   a graph, a description of a relationship, or two input-output pairs (include reading these from a
386   table). ★
387   3.   Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a
388   quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★

389

390   **Interpret expressions for functions in terms of the situation they model.**

391   5.   Interpret the parameters in a linear or exponential function in terms of a context. ★ [Linear and
392   exponential of form f(x)=b^x+k.]

393   **6.   Apply quadratic functions to physical problems, such as the motion of an object under the**
394   **force of gravity. CA ★**

395

396   Modeling the world often involves investigating rates of change, patterns of growth.

397   Two important families of functions characterized by laws of growth are linear functions,

398   which grow at a constant rate, and exponential functions, which grow at a constant

399   percent rate. In standards F-LE.1a-c, students recognize and understand the defining

400   characteristics of linear, quadratic, and exponential functions. Students have already

401   worked extensively with linear equations. They have developed an understanding that

402   an equation in two variables of the form $y = mx + b$ exhibits a special relationship

403   between the variables x and y, namely, that a change of Δx in the variable x, the

404   independent variable, results in a change of $\Delta y = m \cdot \Delta x$ in the dependent variable y.

405   They have seen this informally, in graphs and tables of linear relationships, starting in

406   the grade eight standards (8.EE.5, 8.EE.6, 8.F.3). For example, students recognize that

407   for successive whole number input values, x and $x + 1$, a linear function f, defined by

408     $f(x) = mx + b$, exhibits a constant rate of change:

$$f(x + 1) - f(x) = [m(x + 1) + b] - (mx + b) = m(x + 1 - x) = m.$$

409   Standard F-LE.1a requires students to prove that linear functions exhibit such growth

410   patterns.

411

412   In contrast, an exponential function exhibits a constant percent change

413   in the sense that such functions exhibit a constant *ratio* between output values for

414   successive input values.[1] For instance, a t-table for the equation $y = 3^x$, illustrates the

415   constant ratio of successive y-values for this equation:

x	$y = 3^x$	Ratio of successive y-values
1	3	
2	9	9/3=3
3	27	27/9=3
4	81	81/27=3

416

---

[1] Note: In CA CCSSM Algebra I, only integer values for x are considered in exponential equations such as $y = b^x$.

417   In general, a function g, defined by $g(x) = ab^x$, can be shown to exhibit this constant

418   ratio growth pattern:

$$\frac{g(x+1)}{g(x)} = \frac{ab^{x+1}}{ab^x} = \frac{b^{x+1}}{b^x} = b^{(x+1)-x} = b.$$

419   In Algebra I, students are not required to prove that exponential functions exhibit this

420   growth rate, however, they must be able to recognize situations that represent both

421   linear and exponential functions and construct functions to describe the situations (F-

422   LE.2). Finally, students interpret the *parameters* in linear, exponential and quadratic

423   expressions, and model physical problems with such functions.   When presented in a

424   modeling situation, often the meaning of parameters becomes much clearer than when

425   presented abstractly.

426

427   A graphing utility, spreadsheet, or computer algebra system can be used to experiment

428   with properties of these functions and their graphs and to build computational models of

429   functions, including recursively defined functions (MP.5). A real-world example where

430   this can be explored involves investments, mortgages, half-lives of pharmaceuticals and

431   other financial instruments. For example, students can develop formulas for annual

432   compound interest based on a general formula, such as $P(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt}$, where r is

433   the interest rate, n is the number of times the interest is compounded per year, and t is

434   the number of years the money is invested. They can explore values after different time

435   periods and compare different rates and plans using computer algebra software or

436   simple spreadsheets (MP.5). This hands-on experimentation with such functions helps

437   develop an understanding of their behavior.

438

439   **Conceptual Category: Number and Quantity**

440   In the grade eight standards, students encountered some examples of irrational

441   numbers, such as π and $\sqrt{2}$ (or $\sqrt{n}$ for n a nonsquare number). In Algebra I, students

442   extend this understanding beyond the fact that there are numbers that are not rational;

443   they begin to understand that the rational numbers form a closed system themselves.

444   Students have witnessed that with each extension of number, the meanings of addition,

445   subtraction, multiplication, and division are extended. In each new number system—

446   whole numbers, rational numbers, and real numbers— the distributive law continues to

447   hold, and the commutative and associative laws are still valid for both addition and

448   multiplication. However, in Algebra I students go further along this path.

449

450   **The Real Number System**   **N-RN**

451   **Extend the properties of exponents to rational exponents.**
452   1.   Explain how the definition of the meaning of rational exponents follows from extending the
453   properties of integer exponents to those values, allowing for a notation for radicals in terms of
454   rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want*
455   *$(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
456   2.   Rewrite expressions involving radicals and rational exponents using the properties of exponents.
457
458   **Use properties of rational and irrational numbers.**
459   3.   Explain why the sum or product of two rational numbers is rational; that the sum of a rational
460   number and an irrational number is irrational; and that the product of a nonzero rational number
461   and an irrational number is irrational.
462

463   With N-RN.1, students make meaning of the representation of radicals with rational

464   exponents. Students are first introduced to exponents in grade six; by now, they should

465   have an understanding of the basic properties of exponents (e.g. that $x^n \cdot x^m =$

466     $x^{n+m}, (x^n)^m = x^{nm}, \frac{x^n}{x^m} = x^{n-m}, x^0 = 1$ for $x \neq 0$, etc.). In fact, they may have justified

467   certain properties of exponents by reasoning with other properties (MP.3, MP.7), for

468   example, justifying why any nonzero number to the power 0 is equal to 1:

$$x^0 = x^{n-n} = \frac{x^n}{x^n} = 1, \text{ for } x \neq 0.$$

469   They further their understanding of exponents in Algebra I by using these properties to

470   explain the meaning of rational exponents. For example, properties of whole-number

471   exponents suggest that $(5^{1/3})^3$ should be the same as $5^{[(1/3)\cdot 3]} = 5^1 = 5$, so that $5^{1/3}$

472   should represent the cube root of 5. In addition, the fact that $(ab)^n = a^n \cdot b^n$ reveals

473   that

$$\sqrt{20} = (4 \cdot 5)^{\frac{1}{2}} = 4^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5},$$

474   showing that $\sqrt{20} = 2\sqrt{5}$. The intermediate steps of writing the square root as a rational

475   exponent are necessary at first, but eventually students can work more quickly,

476   understanding the reasoning underpinning this process. Students extend such work

477   with radicals and rational exponents to variable expressions as well, e.g. rewriting an

478   expression like $(a^2 b^5)^{3/2}$ using radicals (N-RN.2).

479

480   In standard N-RN.3, students explain that the sum or product of two rational numbers is

481   rational, by arguing that the sum of two fractions with integer numerator and

482   denominator is also a fraction of the same type, showing that the rational numbers are

483   *closed* under the operations of addition and multiplication (MP.3). The notion that this

484   set of numbers is closed under these operations will be extended to the sets of

485   polynomials and rational functions in later courses. Moreover, students argue that the

486   sum of a rational and irrational is irrational, and the product of a non-zero rational and

487   an irrational is still irrational, showing that the irrational numbers are truly another

488   unique set of numbers that along with the rational numbers forms a larger *system*, the

489   system of real numbers (MP3, MP7).

490

491   **Quantities**   **N-Q**

492   **Reason quantitatively and use units to solve problems.** [Foundation for work with expressions, equations and
493   functions.]

494   1.   Use units as a way to understand problems and to guide the solution of multi-step problems;
495   choose and interpret units consistently in formulas; choose and interpret the scale and the origin
496   in graphs and data displays. ★
497   2.   Define appropriate quantities for the purpose of descriptive modeling. ★
498   3.   Choose a level of accuracy appropriate to limitations on measurement when reporting
499   quantities.★
500

501   In real-world problems, the answers are usually not pure numbers but *quantities*:

502   numbers with units, which involves measurement. In their work in measurement up

503   through grade eight, students primarily measure commonly used attributes such as

504   length, area, and volume. In higher mathematics, students encounter a wider variety of

505   units in modeling, e.g., when considering acceleration, currency conversions, derived

506   quantities such as person-hours and heating degree-days, social science rates such as

507   per-capita income, and rates in everyday life such as points scored per game or batting

508   averages.

509

510   In Algebra I, students reason through problems with careful selection of units, and they

511   use units to understand problems and make sense of the answers they deduce. The

512   following example illustrates the facility with units that students are expected to attain in

513   this domain.

514

Example N-Q: As Felicia gets on the freeway to drive to her cousin's house, she notices that she is a little low on gas. There is a gas station at the exit she normally takes, and she wonders if she will have to get gas before then. She normally sets her cruise control at the speed limit of 70 mph and the freeway portion of the drive takes about an hour and 15 minutes. Her car gets about 30 miles per gallon on the freeway, and gas costs $3.50 per gallon. a. Describe an estimate that Felicia might do in her head while driving to decide how many	**Solution:** a. To estimate the amount of gas she needs, Felicia calculates the distance traveled at 70 mph for 1.25 hours. She might calculate $70 \cdot 1.25 = 70 + 0.25 \cdot 70 = 70 + 17.5 = 87.5$ miles. Since 1 gallon of gas will take her 30 miles, 3 gallons of gas will take her 90 miles, a little more than she needs. So she might figure that 3 gallons is enough. b. Since Felicia pays $3.50 for one gallon of gas, and one gallon of gas takes her 30 miles, it costs her $3.50 to travel 30 miles, therefore

gallons of gas she needs to make it to the gas station at the other end. b. Assuming she makes it, how much does Felicia spend per mile on the freeway?	$\dfrac{\$3.50}{30 \text{ miles}} \approx \dfrac{\$0.12}{1 \text{ mile}}$, meaning it costs Felicia 12 cents to travel each mile on the freeway.

515

516   **Conceptual Category: Algebra**

517

518   In the Algebra conceptual category, students extend the work with expressions they

519   started in the middle grades standards. They create and solve equations in context,

520   utilizing the power of variable expressions to model real-world problems and solve them

521   with attention to units and the meaning of the answers they obtain. They continue to

522   graph equations, understanding the resulting picture as a representation of the points

523   satisfying the equation. This conceptual category comprises a large portion of the

524   Algebra I course and along with the Functions category represents the main body of

525   content.

526

527   The Algebra category in higher mathematics is very closely allied with the Functions

528   category:

529   • An expression in one variable can be viewed as defining a function: the act of

530   evaluating the expression is an act of producing the function's output given

531   the input.

532   • An equation in two variables can sometimes be viewed as defining a function,

533   if one of the variables is designated as the input variable and the other as the

534   output variable, and if there is just one output for each input. This is the case

535   if the expression is of the form $y = (\text{expression in } x)$ or if it can be put into that

536   form by solving for y.

537   • The notion of equivalent expressions can be understood in terms of functions:

538   if two expressions are equivalent they define the same function.

539   • The solutions to an equation in one variable can be understood as the input

540   values that yield the same output in the two functions defined by the

541   expressions on each side of the equation. This insight allows for the method

542   of finding approximate solutions by graphing functions defined by each side

543   and finding the points where the graphs intersect.

544   Thus, in light of understanding functions, the main content of the Algebra category,

545   solving equations, working with expressions, etc., has a very important purpose.

546

547   **Seeing Structure in Expressions**   **A-SSE**

548   **Interpret the structure of expressions.** [Linear, exponential, and quadratic.]
549   1.   Interpret expressions that represent a quantity in terms of its context. ★
550   a.   Interpret parts of an expression, such as terms, factors, and coefficients. ★
551   b.   Interpret complicated expressions by viewing one or more of their parts as a single entity. *For*
552   *example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* ★
553   2.   Use the structure of an expression to identify ways to rewrite it.
554

555   An expression can be viewed as a recipe for a calculation, with numbers, symbols that

556   represent numbers, arithmetic operations, exponentiation, and, at more advanced

557   levels, the operation of evaluating a function. Conventions about the use of parentheses

558   and the order of operations assure that each expression is unambiguous. Creating an

559   expression that describes a computation involving a general quantity requires the ability

560   to express the computation in general terms, abstracting from specific instances.

561

562   Reading an expression with comprehension involves analysis of its underlying structure.

563   This may suggest a different but equivalent way of writing the expression that exhibits

564   some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the

565   addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax

566   is the same as multiplying the price by a constant factor. Students began this work in

567   grades six and seven, and continue this work with more complex expressions.

568

569   **Write expressions in equivalent forms to solve problems.** [Quadratic and exponential.]
570   3.   Choose and produce an equivalent form of an expression to reveal and explain properties of the
571   quantity represented by the expression.   ★
572   a.   Factor a quadratic expression to reveal the zeros of the function it defines.   ★
573   b.   Complete the square in a quadratic expression to reveal the maximum or minimum value of
574   the function it defines.   ★
575   c.   Use the properties of exponents to transform expressions for exponential functions. *For*
576   *example, the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the*
577   *approximate equivalent monthly interest rate if the annual rate is 15%.*★
578

579   In Algebra I, students work with examples of more complicated expressions, such as

580   those that involve multiple variables and exponents. Students use the distributive

581   property to investigate equivalent forms of quadratic expressions, e.g. by writing

582    $(x + y)(x - y) = x(x - y) + y(x - y) = x^2 - xy + xy - y^2 = x^2 - y^2$, thereby yielding a

583   special case of a factorable quadratic, the difference of squares. Students factor

584   second-degree polynomials by making use of such special forms, and by using factoring

585   techniques based on properties of operations (A-SSE.2, A-SSE.2.1). Note that the

586   standards avoid talking about "simplification," because it is often not clear what the

587   simplest form of an expression is, and even in cases where it is clear, it is not obvious

588   that the simplest form is desirable for a given purpose. The standards emphasize

589   purposeful transformation of expressions into equivalent forms that are suitable for the

590   purpose at hand, as the example below shows.

591

Example: *Which is the simpler form?* A particularly rich mathematical investigation involves finding a general expression for the sum of the first n consecutive natural numbers: $$S = 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n.$$ A famous tale speaks of a young C.F. Gauss being able to add the first 100 natural numbers quickly in his head, wowing his classmates and teachers alike. One way to find this sum is to consider the "reverse" of the sum: $$S = n + (n-1) + (n-2) + \cdots + 3 + 2 + 1,$$ and to then add the two expressions for S together,	obtaining: $$2S = (n+1) + (n+1) + \cdots +$$ $$(n+1) + (n+1) + (n+1),$$ where there are n terms of the form $(n+1)$. Thus, $2S = n(n+1)$, so that $S = n(n+1)/2$. While students may be tempted to transform this expression into $\frac{1}{2}n^2 + \frac{1}{2}n$, they are obscuring the ease with which they can evaluate the first expression. Indeed, since n is a natural number, one of either n or $n+1$ is even, so evaluating $n(n+1)/2$, especially mentally, is often easier. Indeed, in Gauss' case, $\frac{100(101)}{2} = 50(101) = 5050$.

592

593 Students also use different forms of the same expression to reveal important

594 characteristics of the expression. For instance, when working with quadratics, they

595 complete the square in the expression $x^2 - 3x + 4$ to obtain the equivalent expression

596 $\left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$. Students can then reason with the new expression that the term being

597 squared is always greater than or equal to 0; hence, the value of the expression will

598 always be greater than or equal to $\frac{7}{4}$ (A-SSE.3, MP.3). A spreadsheet or a computer

599 algebra system (CAS) can be used to experiment with algebraic expressions, perform

600 complicated algebraic manipulations, and understand how algebraic manipulations

601 behave, further contributing to students' understanding of work with expressions (MP.5).

602

603 **Arithmetic with Polynomials and Rational Expressions**   **A-APR**
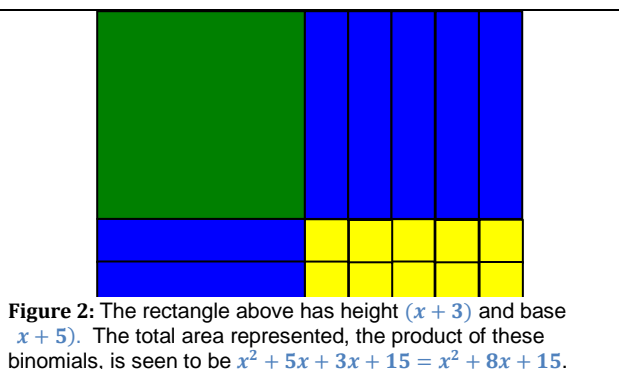
604 **Perform arithmetic operations on polynomials.** [Linear and quadratic.]

605 1.   Understand that polynomials form a system analogous to the integers, namely, they are closed
606   under the operations of addition, subtraction, and multiplication; add, subtract, and multiply
607   polynomials**.**

608

609   In Algebra I, students begin to explore the set of polynomials in x as a system in its own

610   right, subject to certain operations and properties. To perform operations with

611   polynomials meaningfully, students are encouraged to draw parallels between the set of

612   integers, wherein integers can be added, subtracted, and multiplied according to certain

613   properties, and the set of all polynomials with real coefficients (A-APR.1, MP.7). If the

614   function concept is developed before or concurrently with the study of polynomials, then

615   a polynomial can be identified with the function it defines. In this way, $x^2 - 2x - 3$,

616    $(x + 1)(x - 3)$, and $(x - 1)^2 - 4$ are all the same polynomial because they all define the

617   same function.

618

619   In Algebra I, students are only required to add linear or quadratic polynomials and to

620   multiply linear polynomials to obtain quadratic ones, since in later courses they will

621   explore polynomials of higher degree. Students fluently add, subtract, and multiply

622   linear expressions of the form $ax + b$, and add and subtract expressions of the form

623    $ax^2 + bx + c$, with a, b and c real numbers, understanding that the result is yet another

624   expression of one of these forms. The explicit notion of *closure* of the set of

625   polynomials need not be explored in Algebra I.

626

Understanding addition and subtraction of polynomials and the multiplication of monomials and binomials can be supported using manipulatives, such as "algebra tiles," which can offer a concrete representation of the terms in a polynomial (MP.5). The tile representation relies on the *area interpretation of multiplication*: the notion that the product ab can be thought of as the area of

Figure 2: The rectangle above has height $(x + 3)$ and base $x + 5)$. The total area represented, the product of these binomials, is seen to be $x^2 + 5x + 3x + 15 = x^2 + 8x + 15$.

a rectangle of dimensions a units and b units. With this understanding, tiles can be used to represent 1 square unit (a 1×1 tile), x square units (a $1 \times x$ tile), and x^2 square units (an $x \times x$ tile). Finding the product $(x + 5)(x + 3)$ amounts to finding the area of an abstract rectangle of dimensions $x + 5$ and $x + 3$ as illustrated in the figure (MP.2).	Care must be taken in the way negative numbers are handled with this representation. The tile representation of polynomials is also very useful for understanding the notion of completing the square, as described below.

627

628

Creating Equations   A-CED

Create equations that describe numbers or relationships. [Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only.]

1.   Create equations and inequalities in one variable **including ones with absolute value** and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **CA★**
2.   Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
3.   Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★
4.   Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* ★

An equation is a statement of equality between two expressions. The values that make

the equation true are the solutions to the equation. An identity, in contrast, is true for all

values of the variables; rewriting an expression in an equivalent form often creates

identities. The solutions of an equation in one variable form a set of numbers; the

solutions of an equation in two variables form a set of ordered pairs of numbers, which

can be plotted in the coordinate plane. In this set of standards, students create

equations to solve problems, they correctly graph the equations on coordinate axes,

and they interpret solutions in a modeling context. The example below requires

students to understand the multiple variables that appear in a given equation and to

reason with them.

| **Example:** The height h of a ball at time t seconds thrown vertically upward at a speed of v ft/sec is given by the equation $$h = 6 + vt - 16t^2$$ Write an equation whose solution is

 a. The time it takes a ball thrown at a speed of 88 ft/sec to rise 20 feet.

 b. The speed with which the ball must be thrown to rise 20 feet in 2 seconds. | **Solution and Comments:**

 a. We want $h = 20$, and we are told that $v = 88$, so the equation is $20 = 6 + 88t - 16t^2$.

 b. We want $h = 20$, and we are told that $t = 2$, so the equation is $20 = 6 + 2v - 16 \cdot 2^2$.

 While this is a straightforward example, students must be able to flexibly see some of the variables in the equation as constants when others are given values. It also does not explicitly state $v = 88$ or $t = 2$, so that students must understand the meaning of the variables in order to proceed with the problem (MP.1). |

654

655   One change in the CA CCSSM is the creation of equations involving absolute values (A-

656   CED.1). The basic absolute value function has at least two very useful definitions, a

657   descriptive, verbal definition, and a formula definition. A common definition of the

658   absolute value of x is:

659                    $|x|$ = the distance from the number x to 0 (on a number line).

660   An understanding of the number line easily yields that, for example, $|0| = 0$, $|7| = 7$, and

661    $|-3.9| = 3.9$. However, an equally valid "formula" definition of absolute value reads:

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

662   In other words, $|x|$ is simply x whenever x is 0 or positive, but $|x|$ is the opposite of x

663   whenever x is negative. Either definition can be extended to an understanding of the

664   expression $|x - a|$ as the distance between x and a on a number line, an interpretation

665   that has many uses. For a simple application of this idea, suppose a certain bolt is to

666   be mass-produced in a factory with the specification that its width should be 5mm with

667   an error no larger than 0.01mm. If w represents the width of a given bolt produced on

668   the production line, then we want w to satisfy the inequality $|w - 5| \leq 0.01$, i.e., the

669   difference between the actual width w and the target width should be less than or equal

670   to 0.01 (MP.4, MP.6). Students should become comfortable with the basic properties of

671   absolute values (e.g. $|x| + a \neq |x + a|$ in general etc.) and with solving absolute value

672   equations and interpreting the solution.

In higher math courses intervals on the number line are often denoted by an inequality of the form $|x - a| \leq d$ for a positive number d. For example $|x - 2| \leq \frac{1}{2}$ represents the closed interval $1\frac{1}{2} \leq x \leq 2\frac{1}{2}$. This can be seen by interpreting $|x - 2| \leq \frac{1}{2}$ as "the distance from x to 2 is less than or equal to $\frac{1}{2}$" and deciding which numbers fit this description.	On the other hand, in the case that $x - 2 < 0$, we would have $|x - 2| = -(x - 2) \leq \frac{1}{2}$, so that $x \geq 1\frac{1}{2}$. In the case that $x - 2 \geq 0$, we have $|x - 2| = x - 2 \leq \frac{1}{2}$ which means that $x \leq 2\frac{1}{2}$. Since we are looking for all x that satisfy both inequalities, the interval is $1\frac{1}{2} \leq x \leq 2\frac{1}{2}$. This shows how the formula definition can be used to find this interval.

673
674
675   **Reasoning with Equations and Inequalities**   **A-REI**

676   **Understand solving equations as a process of reasoning and explain the reasoning.** [Master linear;
677   learn as general principle.]

678   1.   Explain each step in solving a simple equation as following from the equality of numbers asserted
679   at the previous step, starting from the assumption that the original equation has a solution.
680   Construct a viable argument to justify a solution method.

681

682   **Solve equations and inequalities in one variable.** [Linear inequalities; literal equations that are linear in the variables
683   being solved for; quadratics with real solutions.]

684   3.   Solve linear equations and inequalities in one variable, including equations with coefficients
685   represented by letters.
686   **3.1   Solve one-variable equations and inequalities involving absolute value, graphing the**
687   **solutions and interpreting them in context. CA**
688   4.   Solve quadratic equations in one variable.
689   a. Use the method of completing the square to transform any quadratic equation in *x* into an
690   equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula
691   from this form.
692   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing
693   the square, the quadratic formula, and factoring, as appropriate to the initial form of the
694   equation. Recognize when the quadratic formula gives complex solutions and write them as *a*
695   ± *bi* for real numbers *a* and *b*.

696

697   An equation can often be solved by successively deducing from it one or more simpler

698   equations. For example, one can add the same constant to both sides without changing

699   the solutions, but squaring both sides might lead to extraneous solutions. Strategic

700   competence in solving includes looking ahead for productive manipulations and

701   anticipating the nature and number of solutions. In Algebra I, students solve linear

702   equations and inequalities in one variable, including ones with absolute values and

703   equations with coefficients represented by letters (A-REI.3, A-REI.3.1). In addition, this

704   is students' first exposure to quadratic equations, and they learn various techniques for

705   solving them and the relationship between those techniques (A-REI.4.a-b). When

706   solving equations, students make use of the symmetric and transitive properties, and

707   certain properties of equality with regards to operations (e.g. "equals added to equals is

708   equal"). Standard A-REI.1 requires that in any situation, students can solve an equation

709   *and explain the steps* as resulting from previous true equations and using the

710   aforementioned properties (MP.3). In this way, the idea of *proof*, while not explicitly

711   named, is given a prominent role in the solving of equations, and the reasoning and

712   justification process is not simply relegated to a future mathematics course.

713

On Solving Equations: A written sequence of steps is code for a narrative line of reasoning that would use words like "if", "then", "for all" and "there exists." In the process of learning to solve equations, students should learn certain "if-then" moves: e.g. "if $x = y$ then $x + c = y + c$ for any c." The danger in learning algebra is that students emerge with nothing but the moves, which may make it difficult to detect incorrect or made-up moves later on. Thus the first requirement in this domain (REI) is that students understand that solving equations is a process of reasoning (A.REI.1).	Fragments of Reasoning
	$$x^2 = 4$$ $$x^2 - 4 = 0$$ $$(x - 2)(x + 2) = 0$$ $$x = 2, -2$$ This sequence of equations is short-hand for a line of reasoning: "*If x is a number whose square is 4, then $x^2 - 4 = 0$, by properties of equality. Since $x^2 - 4 = (x - 2)(x + 2)$ for all numbers, it follows that $(x - 2)(x + 2) = 0$. So either $(x - 2) = 0$, in which case $x = 2$, or $(x + 2) = 0$, in which case $x = -2$.*"

714

715   Students in Algebra I extend their work with exponents to working with quadratic

716   functions and equations that have real roots. To extend their understanding of these

717   quadratic expressions and the functions they define, students investigate properties of

718   quadratics and their graphs in the Functions domain.

719

Example: When solving quadratic equations of the form $(x - p)^2 = q$ in standard A-REI.4.a, students rely on the understanding that they can take square roots of both sides of the equation to obtain, $$\sqrt{(x-p)^2} = \sqrt{q}. \qquad (1)$$ In the case that $\sqrt{q}$ is a real number, we can solve this equation for x. A common mistake is to quickly introduce the symbol $\pm$ here, without understanding where it comes from. Doing so without care often leads to students thinking that $\sqrt{9} = \pm 3$, for example.	Note that the quantity $\sqrt{a^2}$ is simply a when $a \geq 0$ (as in $\sqrt{5^2} = \sqrt{25} = 5$), while $\sqrt{a^2}$ is equal to $-a$ (the opposite of a) when $a < 0$ (as in $\sqrt{(-4)^2} = \sqrt{16} = 4$). But this means that $\sqrt{a^2} = |a|$. Applying this to equation (1) yields $|x - p| = \sqrt{q}$. Solving this simple absolute value equation yields that $x - p = \sqrt{q}$ or $-(x - p) = \sqrt{q}$. This results in the two solutions $p + \sqrt{q}, p - \sqrt{q}$.

720

721   Students also transform quadratic equations into the form $ax^2 + bx + c = 0$, for $a \neq 0$,

722   the *standard form* of a quadratic equation. In some cases, the quadratic expression

723   factors nicely and students can apply the zero product property of the real numbers to

724   solve the resulting equation. The *zero product property* states that for two real numbers

725   m and n, $m \cdot n = 0$ if and only if either $m = 0$ or $n = 0$. Hence, when a quadratic

726   polynomial can be rewritten as $a(x - r)(x - s) = 0$, the solutions can be found by

727   setting each of the linear factors equal to 0 separately, and obtaining the solution set

728    $\{r, s\}$. In other cases, a means for solving a quadratic equation arises by *completing the*

729   *square*. Assuming for simplicity that $a = 1$ in the standard equation above, and that the

730   equation has been rewritten as $x^2 + bx = -c$, we can "complete the square" by adding

731   the square of half the coefficient of the x-term to each side of the equation:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2. \qquad (2)$$

732   The result of this simple step is that the quadratic on the left side of the equation is a

733   perfect square,

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2.$$

734   Thus, we have now converted equation (2) into an equation of the form $(x - p)^2 = q$,

735   namely,

$$\left(x + \frac{b}{2}\right)^2 = -c + \frac{b^2}{4},$$

736   which can be solved by the method described above, as long as the term on the right is

737   positive. The case when $a \neq 1$ can be handled similarly and ultimately results in the

738   familiar quadratic formula. Tile representations of quadratics illustrate that the process

739   of completing the square has a geometric interpretation that explains the origin of the

740   name. Students should be encouraged to explore these representations in order to

741   make sense out of the process of completing the square (MP.1, MP.5). Completing the

742   square is an example of a theme that reoccurs throughout algebra: finding ways of

743   transforming equations into certain standard forms that have the same solutions.

744

Completing the Square: The method of completing the square is a useful skill in Algebra. It is generally used to change a quadratic in standard form, $ax^2 + bx + c$, into one in vertex-form, $a(x - h)^2 + k$. The vertex form can help determine several properties of quadratic functions.	subtract 1 to the quadratic expression: $y = x^2 + 8x + 15 + 1 - 1 = x^2 + 8x + 16 - 1.$ Factoring gives us $y = (x + 4)^2 - 1.$ In the picture, note that the tiles used to represent $x^2 + 8x + 15$ have been rearranged to try to form a square, and that a positive unit tile and a "negative"

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Completing the square also has applications in Geometry (G-GPE.1) and later higher mathematics courses. **Example:** To complete the square for the quadratic $y = x^2 + 8x + 15$, we take half the coefficient of the x-term and square it to yield 16. We realize that we need only to add 1 and	unit tile are added into the picture to "complete the square."

745
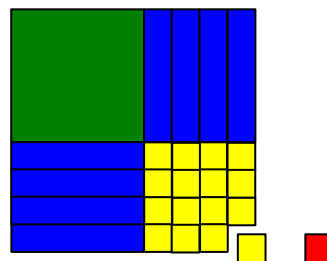
746   The same solution techniques used to solve equations can be used to rearrange

747   formulas to highlight specific quantities and explore relationships between the variables

748   involved. For example, the formula for the area of a trapezoid, $A = \left(\frac{b_1 + b_2}{2}\right) h$, can be

749   solved for *h* using the same deductive process (MP.7, MP.8). As will be discussed

750   later, functional relationships can often be explored more deeply by rearranging

751   equations that define such relationships; thus, the ability to work with equations with

752   letters as coefficients is an important skill.

753

754   **Reasoning with Equations and Inequalities**   **A-REI**

755   **Solve systems of equations.** [Linear-linear and linear-quadratic.]

756   5.   Prove that, given a system of two equations in two variables, replacing one equation by the sum
757   of that equation and a multiple of the other produces a system with the same solutions.
758   6.   Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs
759   of linear equations in two variables.
760   7.   Solve a simple system consisting of a linear equation and a quadratic equation in two variables
761   algebraically and graphically.

762

763   Two or more equations and/or inequalities form a system. A solution for such a system

764   must satisfy every equation and inequality in the system. The process of adding one

765   equation to another is understood as, "if the two sides of one equation are equal, and

766   the two sides of another equation are equal, then the sum (or difference) of the left

767   sides of the two equations is equal to the sum (or difference) of the right sides." The

768   reversibility of these steps justifies that we achieve an equivalent system of equations

769   by doing this. This crucial point should be consistently noted when reasoning about

770   solving systems of equations.

771

| **Example:** *Solving simple systems of equations.* To get started with understanding how to solve systems of equations by linear combinations, students can be encouraged to interpret the system in terms of real-world quantities, at least in some cases. For instance, suppose one wanted to solve the system

$$3x + y = 40$$
$$4x + 2y = 58$$

Now consider the following scenario:

• Suppose 3 CD's and a magazine cost \$40, while 4 CD's and 2 magazines cost \$58. | And the questions

• What happens to the price when you add 1 CD and 1 magazine to your purchase?
• What is the price if you decided to buy only 2 CD's and no magazine?

Answering these questions amounts to realizing that since $(3x + y) + (x + y) = 40 + 18$, we must have that $x + y = 18$. Therefore, $(3x + y) + (-1)(x + y) = 40 + (-1)18$, which implies that $2x = 22$, or 1 CD costs \$11.

The value of y can now be found using either of the original equations: $y = 7$. |

772

773   When solving systems of equations, students also make frequent use substitution, e.g.,

774   when solving the system $2x - 9y = 5$ and $y = \frac{1}{3}x + 1$ by substituting the expression

775    $\frac{1}{3}x + 1$ for y in the first equation, to obtain $2x - 9\left(\frac{1}{3}x + 1\right) = 5$. Students also solve

776   such systems approximately, by using graphs and tables of values (A-REI.5-7).

777

778   **Reasoning with Equations and Inequalities**   **A-REI**
779   **Represent and solve equations and inequalities graphically.** [Linear and exponential; learn as general
780   principle.]
781   10.   Understand that the graph of an equation in two variables is the set of all its solutions plotted in
782   the coordinate plane, often forming a curve (which could be a line).
783   11.   Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and
784            $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately,
785   e.g., using technology to graph the functions, make tables of values, or find successive
786   approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute
787   value, exponential, and logarithmic functions. ★

788   12.   Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary
789   in the case of a strict inequality), and graph the solution set to a system of linear inequalities in
790   two variables as the intersection of the corresponding half-planes.
791

792   One of the most important goals of instruction in mathematics is to illuminate

793   connections between various mathematical concepts. In particular, in standards A-

794   REI.10-12, students learn the relationship between the algebraic representation of an

795   equation and its graph plotted in the coordinate plane, and understand geometric

796   interpretations of solutions to equations and inequalities. In Algebra I students work

797   only with linear, exponential, quadratic, step, piecewise, and absolute value functions.

798   As students become more comfortable with function notation, e.g. writing $f(x) = 3x - 2$

799   and $g(x) = x^2 - x + 2$, they begin to see solving the equation $3x - 2 = x^2 - x + 2$ as

800   solving the equation $f(x) = g(x)$; that is, they are finding those x-values where two

801   functions take on the same output value. Moreover, they graph the two equations, and

802   see that the x-coordinate(s) of the point(s) of intersection of the graphs of $y = f(x)$ and

803   $y = g(x)$ are the solutions to the original equation (shown in Figure 3).

804
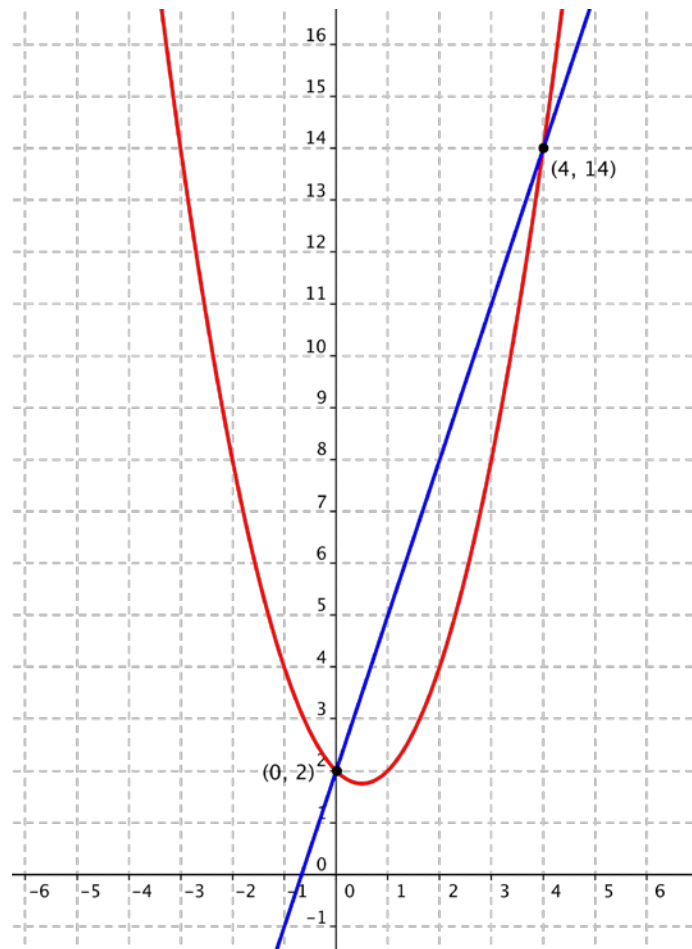
805

806   Figure 3: Graphs of $y = 3x + 2$ and $y = x^2 - x + 2$.

807   Students also create tables of values for functions to approximate or find exact solutions

808   to equations such as that above. For example, they may use spreadsheet software to

809   construct a table such as the one below.

x	f(x)=3x+2	g(x)=x^2-x+2
-6	-16	44
-5	-13	32
-4	-10	22
-3	-7	14
-2	-4	8
-1	-1	4
0	**2**	**2**
1	5	2
2	8	4
3	11	8
4	**14**	**14**
5	17	22
6	20	32
7	23	44
8	26	58
9	29	74

810

811   Figure 4: Table of values for $f(x) = 3x + 2$ and $g(x) = x^2 - x + 2$.

812

813   While a table like this one is in general insufficient proof that all the solutions to a

814   given equation have been found, students can reason in certain situations why they

815   have found all solutions (MP.3, MP.6). In this example, since the original equation is of

816   degree two, we know that there are at most two solutions, so that the solution set is

817   {0,4}.

818

819   **Conceptual Category: Statistics and Probability**

820   In Algebra I, students build on their understanding of key ideas for describing

821   distributions—shape, center, and spread—presented in the standards for grades six

822   through eight. This enhanced understanding allows them to give more precise answers

823   to deeper questions, often involving comparisons of data sets.

824

825   **Interpreting Categorical and Quantitative Data**   **S-ID**

826   **Summarize, represent, and interpret data on a single count or measurement variable.**

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827   1.   Represent data with plots on the real number line (dot plots, histograms, and box plots). ★
828   2.   Use statistics appropriate to the shape of the data distribution to compare center (median, mean)
829   and spread (interquartile range, standard deviation) of two or more different data sets. ★
830   3.   Interpret differences in shape, center, and spread in the context of the data sets, accounting for
831   possible effects of extreme data points (outliers). ★
832
833
834   **Summarize, represent, and interpret data on two categorical and quantitative variables.** [Linear focus,
835   discuss general principle.]
836   5.   Summarize categorical data for two categories in two-way frequency tables. Interpret relative
837   frequencies in the context of the data (including joint, marginal, and conditional relative
838   frequencies). Recognize possible associations and trends in the data. ★
839   6.   Represent data on two quantitative variables on a scatter plot, and describe how the variables are
840   related. ★
841   a.   Fit a function to the data; use functions fitted to data to solve problems in the context of the
842   data. *Use given functions or choose a function suggested by the context. Emphasize linear,*
843   *quadratic, and exponential models.* ★
844   b.   Informally assess the fit of a function by plotting and analyzing residuals. ★
845   c.   Fit a linear function for a scatter plot that suggests a linear association. ★
846
847   **Interpret linear models.**
848   7.   Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the
849   context of the data. ★
850   8.   Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
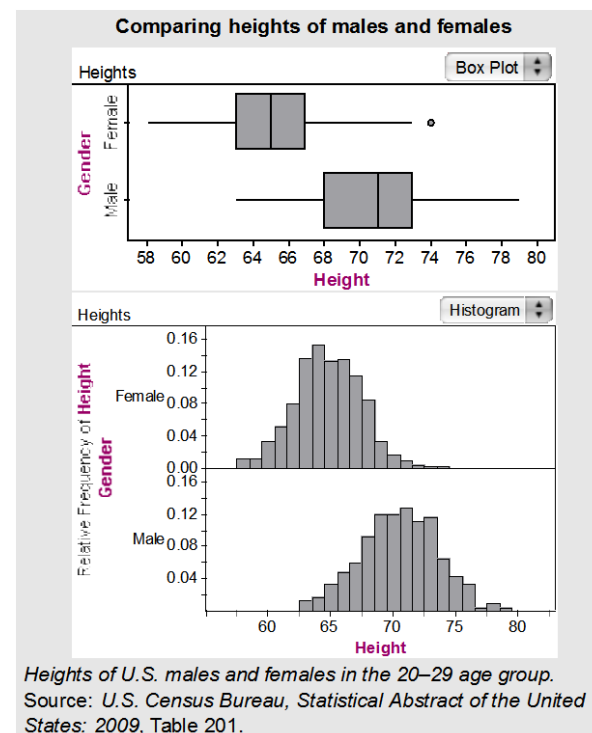851   9.   Distinguish between correlation and causation. ★
852

853   Standards S.ID.1-6 can be considered supporting standards with regards to Standards

854   S.ID.7-9, in the sense that they extend concepts students began learning in grades six

855   through eight. In general, students use shape and the question(s) to be answered to

856   decide on the median or mean as the more appropriate measure of center and to justify

857   their choice through statistical reasoning. Students may use parallel box-plots or

858   histograms to compare differences in the shape, center and spread of comparable data

859   sets (S-ID.1, 2).

860

Example. The graphs below show two ways of comparing height data for males and females in the 20-29 age group. Both involve plotting the data or data summaries (box plots or histograms) on the same scale, resulting in what are called parallel (or	questions about it just from knowledge of these three facts (shape, center, and spread). For either group, about 68% of the data values will be within one standard deviation of the mean (S.ID.2, S.ID.3). Students also observe that the two

side-by-side) box plots and parallel histograms (S-ID.1). The parallel box plots show an obvious difference in the medians and the interquartile ranges (IQRs) for the two groups; the medians for males and females are, respectively, 71 inches and 65 inches, while the IQRs are 4 inches and 5 inches. Thus, male heights center at a higher value but are slightly more variable.

The parallel histograms show the distributions of heights to be mound shaped and fairly symmetrical (approximately normal) in shape. Therefore, the data can be succinctly described using the mean and standard deviation. Heights for males and females have means of 70.4 inches and 64.7 inches, respectively, and standard deviations of 3.0 inches and 2.6 inches. Students should be able to sketch each distribution and answer
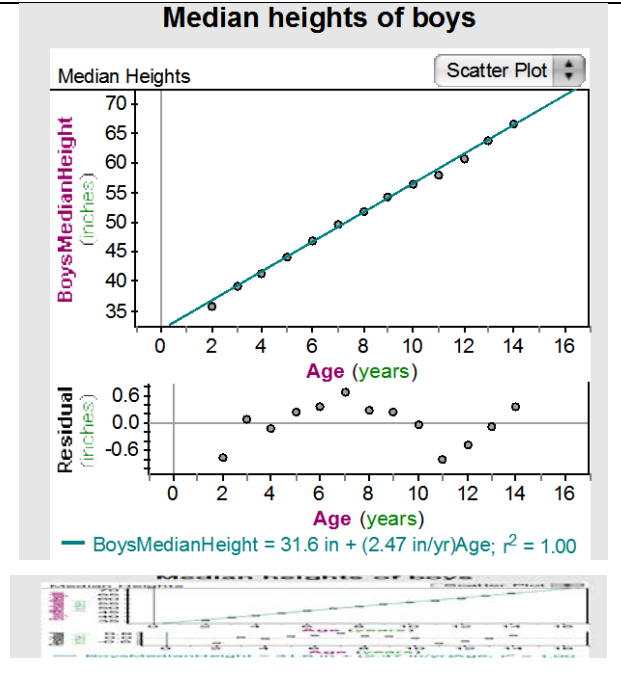
measures of center, median and mean, tend to be close to each other for symmetric distributions.

Heights of U.S. males and females in the 20–29 age group.
Source: *U.S. Census Bureau, Statistical Abstract of the United States: 2009*, Table 201.

861

862 As with univariate data analysis, students now take a deeper look at bivariate data,

863 using their knowledge of proportions to describe categorical associations and using their

864 knowledge of functions to fit models to quantitative data. (S-ID-5, 6) Students have

865 seen scatter plots in the grade eight standards and now extend that knowledge to fit

866 mathematical models that capture key elements of the relationship between two

867 variables and to explain what the model tells us about the relationship. Students must

868 learn to take a careful look at scatter plots, as sometimes the "obvious" pattern does not

869 tell the whole story and may be misleading. A line of best fit may appear to fit data

870 almost perfectly, while an examination of the *residuals*, the collection of differences

871 between corresponding coordinates on a least squares line and the actual data value

872 for a variable may reveal more about the behavior of the data.

873

Example. Students must learn to take a careful look at scatter plots, as sometimes the "obvious" pattern does not tell the whole story, and can even be misleading. The graphs show the median heights of growing boys through the ages 2 to 14. The line (least squares regression line) with slope 2.47 inches per year of growth looks to be a perfect fit (S-ID.6c). But, the *residuals*, the differences between the corresponding coordinates on the least squares line and the actual data values for each age, reveal additional information. A plot of the residuals shows that growth at a constant rate does not proceed at a constant rate over those years (S.ID.6b).	**Median heights of boys** Median Heights Scatter Plot BoysMedianHeight (inches) vs Age (years) Residual (inches) vs Age (years) — BoysMedianHeight = 31.6 in + (2.47 in/yr)Age; r^2 = 1.00

874

875   Finally, students extend their work from topics covered in the grade eight standards and

876   other topics in Algebra I to interpret the parameters of a linear model in the context of

877   data that it represents. They compute *correlation coefficients* using technology and

878   interpret the value of the coefficient (MP.4, MP.5). Students see situations where

879   correlation and causation are mistakenly interchanged, and they are careful to closely

880   examine the story that data and computed statistics are trying to tell. (S-ID.7-9).

881

882   Algebra I Overview

883

884   **Number and Quantity**

885   **The Real Number System**

886   • Extend the properties of exponents to rational exponents.

887   • Use properties of rational and irrational numbers.

888   **Quantities.**

889   • Reason quantitatively and use units to solve problems.

890

891   **Algebra**

892   **Seeing Structure in Expressions**

893   • Interpret the structure of expressions.

894   • Write expressions in equivalent forms to solve problems.

895   **Arithmetic with Polynomials and Rational Expressions**

896   • Perform arithmetic operations on polynomials.

897   **Creating Equations**

898   • Create equations that describe numbers or relationships.

899   **Reasoning with Equations and Inequalities**

900   • Understand solving equations as a process of reasoning and explain the reasoning.

901   • Solve equations and inequalities in one variable.

902   • Solve systems of equations.

903   • Represent and solve equations and inequalities graphically.

904

905   **Functions**

906   **Interpreting Functions**

907   • Understand the concept of a function and use function notation.

908   • Interpret functions that arise in applications in terms of the context.

909   • Analyze functions using different representations.

910   **Building Functions**

911   • Build a function that models a relationship between two quantities.

912   • Build new functions from existing functions.

913   **Linear, Quadratic, and Exponential Models**

914   • Construct and compare linear, quadratic, and exponential models and solve problems.

915   • Interpret expressions for functions in terms of the situation they model.

916

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

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917

918

Statistics and Probability

Interpreting Categorical and Quantitative Data

921 • Summarize, represent, and interpret data on a single count or measurement variable.

922 • Summarize, represent, and interpret data on two categorical and quantitative variables.

923 • Interpret linear models.

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958 ★ Indicates a modeling standard linking mathematics to everyday life, work, and decision-
959 making
960 (+) Indicates additional mathematics to prepare students for advanced courses.

961

962   Algebra I

963

964   **Number and Quantity**

965   **The Real Number System**   **N-RN**

966   **Extend the properties of exponents to rational exponents.**

967   1.   Explain how the definition of the meaning of rational exponents follows from extending the
968   properties of integer exponents to those values, allowing for a notation for radicals in terms of
969   rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want*
970   *$(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
971   2.   Rewrite expressions involving radicals and rational exponents using the properties of exponents.

972

973   **Use properties of rational and irrational numbers.**

974   3.   Explain why the sum or product of two rational numbers is rational; that the sum of a rational
975   number and an irrational number is irrational; and that the product of a nonzero rational number
976   and an irrational number is irrational.

977

978   **Quantities**   **N-Q**

979   **Reason quantitatively and use units to solve problems.** [Foundation for work with expressions, equations and
980   functions.]

981   1.   Use units as a way to understand problems and to guide the solution of multi-step problems;
982   choose and interpret units consistently in formulas; choose and interpret the scale and the origin
983   in graphs and data displays. ★
984   2.   Define appropriate quantities for the purpose of descriptive modeling. ★
985   3.   Choose a level of accuracy appropriate to limitations on measurement when reporting
986   quantities.★

987

988   **Algebra**

989   **Seeing Structure in Expressions**   **A-SSE**

990   **Interpret the structure of expressions.** [Linear, exponential, and quadratic.]

991   1.   Interpret expressions that represent a quantity in terms of its context. ★
992   a.   Interpret parts of an expression, such as terms, factors, and coefficients. ★
993   b.   Interpret complicated expressions by viewing one or more of their parts as a single entity. *For*
994   *example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* ★
995   2.   Use the structure of an expression to identify ways to rewrite it.

996

997   **Write expressions in equivalent forms to solve problems.** [Quadratic and exponential.]

998   3.   Choose and produce an equivalent form of an expression to reveal and explain properties of the
999   quantity represented by the expression.   ★
1000   a.   Factor a quadratic expression to reveal the zeros of the function it defines.   ★
1001   b.   Complete the square in a quadratic expression to reveal the maximum or minimum value of
1002   the function it defines.   ★
1003   c.   Use the properties of exponents to transform expressions for exponential functions. *For*
1004   *example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the*
1005   *approximate equivalent monthly interest rate if the annual rate is 15%.*★
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1009

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| 1010 | **Arithmetic with Polynomials and Rational Expressions** | **A-APR** |

Perform arithmetic operations on polynomials. [Linear and quadratic.]

1011

1012 1. Understand that polynomials form a system analogous to the integers, namely, they are closed
1013   under the operations of addition, subtraction, and multiplication; add, subtract, and multiply
1014   polynomials**.**
1015
1016

| 1017 | **Creating Equations** | **A-CED** |

Create equations that describe numbers or relationships. [Linear, quadratic, and exponential (integer inputs
only); for A.CED.3 linear only.]

1018
1019

1020 1. Create equations and inequalities in one variable **including ones with absolute value** and use
1021   them to solve problems. *Include equations arising from linear and quadratic functions, and simple*
1022   *rational and exponential functions.* **CA★**
1023 2. Create equations in two or more variables to represent relationships between quantities; graph
1024   equations on coordinate axes with labels and scales. ★
1025 3. Represent constraints by equations or inequalities, and by systems of equations and/or
1026   inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For*
1027   *example, represent inequalities describing nutritional and cost constraints on combinations of*
1028   *different foods.* ★
1029 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving
1030   equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.* ★
1031

| 1032 | **Reasoning with Equations and Inequalities** | **A-REI** |

Understand solving equations as a process of reasoning and explain the reasoning. [Master linear;
learn as general principle.]

1033
1034

1035 1. Explain each step in solving a simple equation as following from the equality of numbers asserted
1036   at the previous step, starting from the assumption that the original equation has a solution.
1037   Construct a viable argument to justify a solution method.
1038

Solve equations and inequalities in one variable. [Linear inequalities; literal equations that are linear in the variables
being solved for; quadratics with real solutions.]

1039
1040

1041 3. Solve linear equations and inequalities in one variable, including equations with coefficients
1042   represented by letters.
1043 **3.1 Solve one-variable equations and inequalities involving absolute value, graphing the**
1044   **solutions and interpreting them in context. CA**
1045 4. Solve quadratic equations in one variable.
1046   a. Use the method of completing the square to transform any quadratic equation in *x* into an
1047   equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula
1048   from this form.
1049   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing
1050   the square, the quadratic formula, and factoring, as appropriate to the initial form of the
1051   equation. Recognize when the quadratic formula gives complex solutions and write them as *a*
1052 $\pm$ *bi* for real numbers *a* and *b*.
1053

Solve systems of equations. [Linear-linear and linear-quadratic.]

1054

1055 5. Prove that, given a system of two equations in two variables, replacing one equation by the sum
1056   of that equation and a multiple of the other produces a system with the same solutions.
1057 6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs
1058   of linear equations in two variables.

1059   7.   Solve a simple system consisting of a linear equation and a quadratic equation in two variables
1060   algebraically and graphically.
1061
1062   **Represent and solve equations and inequalities graphically.** [Linear and exponential; learn as general
1063   principle.]
1064   10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in
1065   the coordinate plane, often forming a curve (which could be a line).
1066   11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and
1067 $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately,
1068   e.g., using technology to graph the functions, make tables of values, or find successive
1069   approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute
1070   value, exponential, and logarithmic functions. ★
1071   12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary
1072   in the case of a strict inequality), and graph the solution set to a system of linear inequalities in
1073   two variables as the intersection of the corresponding half-planes.
1074

1075   # Functions

1076   ## Interpreting Functions   F-IF

1077   **Understand the concept of a function and use function notation.** [Learn as general principle; focus on linear
1078   and exponential and on arithmetic and geometric sequences]
1079   1.   Understand that a function from one set (called the domain) to another set (called the range)
1080   assigns to each element of the domain exactly one element of the range. If f is a function and x is
1081   an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The
1082   graph of f is the graph of the equation $y = f(x)$.
1083   2.   Use function notation, evaluate functions for inputs in their domains, and interpret statements that
1084   use function notation in terms of a context.
1085   3.   Recognize that sequences are functions, sometimes defined recursively, whose domain is a
1086   subset of the integers. *For example, the Fibonacci sequence is defined recursively by*
1087   *$f(0) = f(1) = 1, f(n + 1) = f(n) + f(n − 1)$ for $n ≥ 1$.*
1088

1089   **Interpret functions that arise in applications in terms of the context.** [Linear, exponential, and quadratic.]
1090   4.   For a function that models a relationship between two quantities, interpret key features of graphs
1091   and tables in terms of the quantities, and sketch graphs showing key features given a verbal
1092   description of the relationship. *Key features include: intercepts; intervals where the function is*
1093   *increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end*
1094   *behavior; and periodicity.* ★
1095   5.   Relate the domain of a function to its graph and, where applicable, to the quantitative relationship
1096   it describes. *For example, if the function h gives the number of person-hours it takes to assemble*
1097   *n engines in a factory, then the positive integers would be an appropriate domain for the function.*
1098   ★
1099   6.   Calculate and interpret the average rate of change of a function (presented symbolically or as a
1100   table) over a specified interval. Estimate the rate of change from a graph. ★
1101

1102   **Analyze functions using different representations.** [Linear, exponential, quadratic, absolute value, step, piecewise-
1103   defined.]
1104   7.   Graph functions expressed symbolically and show key features of the graph, by hand in simple
1105   cases and using technology for more complicated cases.   ★
1106   a.   Graph linear and quadratic functions and show intercepts, maxima, and minima. ★
1107   b.   Graph square root, cube root, and piecewise-defined functions, including step functions and
1108   absolute value functions. ★

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1109   e.   Graph exponential and logarithmic functions, showing intercepts and end behavior, and
1110   trigonometric functions, showing period, midline, and amplitude. ★
1111   8.   Write a function defined by an expression in different but equivalent forms to reveal and explain
1112   different properties of the function.
1113   a.   Use the process of factoring and completing the square in a quadratic function to show zeros,
1114   extreme values, and symmetry of the graph, and interpret these in terms of a context.
1115   b.   Use the properties of exponents to interpret expressions for exponential functions. *For*
1116   *example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$,*
1117   *$y = (1.01)^{12t}$, and $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.*
1118   9.   Compare properties of two functions each represented in a different way (algebraically,
1119   graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one*
1120   *quadratic function and an algebraic expression for another, say which has the larger maximum.*
1121
1122

Building Functions   F-BF

1123

Build a function that models a relationship between two quantities. [For F.BF.1, 2, linear, exponential, and
1124
1125 quadratic.]
1126   2.   Write a function that describes a relationship between two quantities.   ★
1127   a.   Determine an explicit expression, a recursive process, or steps for calculation from a
1128   context.   ★
1129   b.   Combine standard function types using arithmetic operations. *For example, build a function*
1130   *that models the temperature of a cooling body by adding a constant function to a decaying*
1131   *exponential, and relate these functions to the model.* ★
1132   2.   Write arithmetic and geometric sequences both recursively and with an explicit formula, use them
1133   to model situations, and translate between the two forms. ★
1134

Build new functions from existing functions. [Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear
1135
1136 only.]
1137   3.   Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific
1138   values of k (both positive and negative); find the value of k given the graphs. Experiment with
1139   cases and illustrate an explanation of the effects on the graph using technology. *Include*
1140   *recognizing even and odd functions from their graphs and algebraic expressions for them.*
1141   4.   Find inverse functions.
1142   a.   Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an
1143   expression for the inverse.
1144

Linear, Quadratic, and Exponential Models   F-LE

1145

Construct and compare linear, quadratic, and exponential models and solve problems.
1146
1147   1.   Distinguish between situations that can be modeled with linear functions and with exponential
1148   functions. ★
1149   a.   Prove that linear functions grow by equal differences over equal intervals, and that
1150   exponential functions grow by equal factors over equal intervals. ★
1151   b.   Recognize situations in which one quantity changes at a constant rate per unit interval
1152   relative to another. ★
1153   c.   Recognize situations in which a quantity grows or decays by a constant percent rate per unit
1154   interval relative to another. ★
1155   2.   Construct linear and exponential functions, including arithmetic and geometric sequences, given
1156   a graph, a description of a relationship, or two input-output pairs (include reading these from a
1157   table). ★
1158   3.   Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a
1159   quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★
1160

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1161 **Interpret expressions for functions in terms of the situation they model.**

1162 5.   Interpret the parameters in a linear or exponential function in terms of a context. ★ [Linear and
1163   exponential of form $f(x)=b^x+k$.]

1164 **6.   Apply quadratic functions to physical problems, such as the motion of an object under the**
1165 **force of gravity. CA ★**

1166

1167 ## Statistics and Probability

1168 **Interpreting Categorical and Quantitative Data   S-ID**

1169 **Summarize, represent, and interpret data on a single count or measurement variable.**

1170 1.   Represent data with plots on the real number line (dot plots, histograms, and box plots). ★
1171 2.   Use statistics appropriate to the shape of the data distribution to compare center (median, mean)
1172   and spread (interquartile range, standard deviation) of two or more different data sets. ★
1173 3.   Interpret differences in shape, center, and spread in the context of the data sets, accounting for
1174   possible effects of extreme data points (outliers). ★

1175

1176 **Summarize, represent, and interpret data on two categorical and quantitative variables.** [Linear focus,
1177 discuss general principle.]

1178 5.   Summarize categorical data for two categories in two-way frequency tables. Interpret relative
1179   frequencies in the context of the data (including joint, marginal, and conditional relative
1180   frequencies). Recognize possible associations and trends in the data. ★
1181 6.   Represent data on two quantitative variables on a scatter plot, and describe how the variables are
1182   related. ★
1183   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the
1184   data. *Use given functions or choose a function suggested by the context. Emphasize linear,*
1185   *quadratic, and exponential models. ★*
1186   b. Informally assess the fit of a function by plotting and analyzing residuals. ★
1187   c. Fit a linear function for a scatter plot that suggests a linear association. ★

1188

1189 **Interpret linear models.**

1190 7.   Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the
1191   context of the data. ★
1192 8.   Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
1193 9.   Distinguish between correlation and causation. ★

1194